# TLM Simulation of Electromagnetic Wave Propagation in Anisotropic Moving Media

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***Abstract* — A TLM (Transmission Line Matrix) model suitable to simulate the propagation of waves in moving anisotropic continuous media is presented. As it is well known, an electromagnetic wave propagating in a general medium, moving with respect to its source, experiences a drag by the own medium, which involves a wave velocity dependent on the direction of propagation. In this work, we present a first approach for the case of uniform movement of an anisotropic dielectric medium with respect to an electromagnetic source. Although the technique allows for quantitative results, a special attention is devoted to the simulation of wave fronts: distorted elliptical fronts because a particular kind of "anisotropy" appears, even in an isotropic medium.**

## I. INTRODUCTION

The TLM algorithm [1] is strongly dependent on the synchronism of pulse propagation and on the network geometry: in this way, the modelling of complex media is often a very uneasy task. In order to simulate the proposed electromagnetic problem, we start from a previous modelling technique suitable for motionless anisotropic dielectric media: the permittivity tensor of such media means the coupling of electric field vectors components. TLM allows modelling such a coupling by adding new components to the circuit representation of the transmission lines network. This is achieved by adding a voltage source to each node on the mesh [2], accounting for the coupling of electric fields.

The simplest particular cases are the isotropic moving media. The propagation speed in an anisotropic medium at rest depends on the direction, the phenomenon being then different to the one happening in an isotropic moving media. Let's consider a plane wave linearly polarised in an anisotropic medium. Its speed depends on the polarization of its fields. On the contrary, in an isotropic medium moving with respect to the source, [3]–[4], the propagation speed varies in both directions (forward and backward).

As we write below, when the electromagnetic source moves in an anisotropic medium, the two effects of fields coupling are superposed.

## II. PLANE WAVES IN MOVING MEDIA

### *A. The Lorentz Transformation (LT)*

As it is well known from the special relativity, the space and time coordinates for different observers attached to inertial frames, are transformed through the Lorentz transformation (LT), giving rise to the invariance of physical laws. Suppose that for an observer S', the electrodynamics laws are described by the usual Maxwell equations:

$$\nabla\overline{D}'=\rho',\nabla\overline{B}'=0,\nabla\times\overline{E}'=-\frac{\partial\overline{B}'}{\partial t},\nabla\times\overline{H}'=\frac{\partial\overline{D}'}{\partial t}+\overline{J}' \quad (1)$$

For any other observer in an inertial frame S moving with respect to S', the electrodynamic equations are still the same:

$$\nabla\overline{D}=\rho,\ \nabla\overline{B}=0,\ \nabla\times\overline{E}=-\frac{\partial\overline{B}}{\partial t},\ \nabla\times\overline{H}=\frac{\partial\overline{D}}{\partial t}+\overline{J} \quad (2)$$

and the effect of the movement is included in the field transformation and the constitutive relations. As a direct consequence of this invariance, the speed of light $c$ is a constant in both S and S'.

Let's consider the simplest case where both coordinate systems S and S' lies parallels to each other and S' moves uniformly with respect to S with a speed less than $c$, expressed through $\beta$:

$$\overline{v}=\overline{\beta}\,c=\overline{\beta}\big/\sqrt{\varepsilon_o\mu_o} \quad (3)$$

If Maxwell equations are to be invariant under LT, the field vectors must be transformed following, [3]:

$$\left.\begin{aligned}\overline{E}&=\left(\overline{E}'-\overline{\beta}\times c\overline{B}'\right)\big/\sqrt{1-\beta^2}\\ c\overline{D}&=\left(c\overline{D}'-\overline{\beta}\times\overline{H}'\right)\big/\sqrt{1-\beta^2}\\ c\overline{B}&=\left(c\overline{B}'+\overline{\beta}\times\overline{E}'\right)\big/\sqrt{1-\beta^2}\\ \overline{H}&=\left(\overline{H}'+\overline{\beta}\times c\overline{D}'\right)\big/\sqrt{1-\beta^2}\end{aligned}\right\} \quad (4)$$



 

It must be noted that the field components parallel to the direction of relative motion are not transformed at all.

*B. The 2D Wave Equation*

Let's $(\varepsilon)'$ and $(\mu)'$ denote the permittivity and permeability tensor respectively of an anisotropic medium, as measured in the rest frame S':

$$\overline{D}' = (\varepsilon)' \cdot \overline{E}', \ \overline{B}' = (\mu)' \cdot \overline{H}' \quad (5)$$

The observer S establishes then the following equations as constitutive relations :

$$\begin{aligned} \overline{D} + \varepsilon_o \mu_o \ \overline{v} \times \overline{H} &= (\varepsilon)' \overline{E} + (\varepsilon)' \overline{v} \times \overline{B} \\ \overline{B} - \varepsilon_o \mu_o \ \overline{v} \times \overline{E} &= (\mu)' \ \overline{H} - (\mu)' \overline{v} \times \overline{D} \end{aligned} \quad (6)$$

The resulting equations (4) and (6), allows obtaining the wave equation describing the propagation in the frame where the source fields are in rest.

Let's consider the propagation in a non–dispersive anisotropic dielectric medium with its optical axis along the Z–axis. Then, we can write the permittivity and permeability tensor in the XY plane as follows:

$$(\varepsilon)' = \begin{pmatrix} \varepsilon_x & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_y \end{pmatrix} \varepsilon_o, \ (\mu)' = \begin{pmatrix} \mu_r & 0 \\ 0 & \mu_r \end{pmatrix} \mu_o \quad (7)$$

Now, when the source is moving uniformly along the X–axis, as:

$$\overline{\beta} = \beta \hat{u}_x \quad (8)$$

the wave equation for each field component normal to X (lying in the YZ plane) is written in the same way for the observer S', i.e., the field equation for the Z component of the magnetic field strength is:

$$\begin{aligned} &\left(m\varepsilon_x - \mu_r \beta^2\right) \frac{\partial^2 H_z}{\partial x^2} + \left(1 - \beta^2\right) m\varepsilon_y \frac{\partial^2 H_z}{\partial y^2} \\ &+ 2 \frac{\beta}{c} \left(m\varepsilon_x - \mu_r\right) \frac{\partial^2 H_z}{\partial x \partial t} \\ &- \left(2 - \beta^2\right) m\varepsilon_{xy} \frac{\beta}{c} \frac{\partial^2 H_z}{\partial y \partial t} \\ &- \left(2 - \beta^2\right) m\varepsilon_{xy} \frac{\partial^2 H_z}{\partial x \partial y} \\ &= \frac{m\varepsilon_x \beta^2 - \mu_r}{c^2} \frac{\partial^2 H_z}{\partial t^2} \end{aligned} \quad (9)$$

The coefficient $m$ is defined as the inverse of the determinant of relative permittivity tensor in (7).

At this time, we are going to look for the consequences we can get from such equation (9) related to the wave propagation. To do so, we start with the explicit equation for a stationary medium, $\beta = 0$,:

$$\varepsilon_x \frac{\partial^2 H_z}{\partial x^2} + \varepsilon_y \frac{\partial^2 H_z}{\partial y^2} - 2\varepsilon_{xy} \frac{\partial^2 H_z}{\partial x \partial y} = \frac{\mu_r}{mc^2} \frac{\partial^2 H_z}{\partial t^2} \quad (10)$$

In comparing the equations (9) and (10), we find two remarkable points in (9).

Firstly, the second space order space-time derivatives (cross-derivative) accounts for the inertia of the medium. The origin of the wave fronts is dragged in time.

Secondly, the coefficients of the second order time derivative are differently « weighted » in (9) and (10) as the coefficients of the second order space derivative, normal to X, are too. This is equivalent to an « effective elasticity » giving rise to distorted elliptical wave fronts.

## III. TLM Modelling

We will extend the TLM method to simulate the wave propagation when its source moves uniformly with respect to an anisotropic medium. The starting point is to discretize the field equations (either Maxwell's or wave equations) for the field component normal to X, (9), then to compare it with the equations modelled by the TLM algorithm, [2]. The terms in the discretized wave equation having no representation in the mesh equations, will then be accounted with new elements added to the network nodes as voltage sources, its value being updated every time iteration.

*A. Wave propagation in anisotropic moving media.*

Let's go back to the propagation of a wave in an anisotropic uniformly moving medium parallel to X–axis, this is the already presented problem. The X field components are not transformed at all, as shown, then we deal with the other components normal to X. We point, for instance to $H_z$, (9), normal to XY plane.

The starting 2D TLM node is a parallel connection of transmission lines with different characteristic admittances $Y_x$ and $Y_y$, with a permittivity stub with admittance $Y_o$, and a series connected voltage source $V_s$, [2]. Then, the field component $H_z$ is represented in the TLM mesh by the total voltage $V_z$ at the node. From the equations of voltage and current at the TLM node, we can write an equation for the total voltage $V_z$ depending on the voltages at neighboring nodes and the time variation of the source, [5]. In this way the values of the elements of the equivalent circuit are related with the electromagnetic problem through a relation that can be easily found by comparing it with the discretized form of (9).

The results of this modelling are the different values of propagation speeds in the transmission lines, which depends on the rate of the real constitutive parameters to the effectives. The arms and permittivity stub normalized admittances for the new TLM node are:

$$Y_x = \frac{m\varepsilon_x - \mu_r\beta^2}{m\varepsilon_y(1-\beta^2)},\ Y_y = 1 \qquad (11)$$

$$Y_o = 4\left\{\frac{\mu_r - m\varepsilon_x\beta^2}{m\varepsilon_y(1-\beta^2)} - \frac{1}{2}\left(\frac{m\varepsilon_x - \mu_r\beta^2}{m\varepsilon_y(1-\beta^2)} + 1\right)\right\} \qquad (12)$$

The voltage source $V_s(t)$, which includes the cross–derivatives of the total voltage at the node, and is denoted by the symbol $\Delta^2$, is updated each time iteration following the rule, at time step $k+1$:

$$\sqrt{2}\beta\frac{m\varepsilon_x - \mu_r}{m\varepsilon_x\beta^2 - \mu_r}\Delta^2_{xt}\left({}_kV_z\right) + \frac{\beta}{2m\varepsilon_{xy}}\frac{2-\beta^2}{m\varepsilon_x\beta^2 - \mu_r}\Delta^2_{yt}\left({}_kV_z\right) + \frac{1}{2m\varepsilon_{xy}}\frac{2-\beta^2}{m\varepsilon_x\beta^2 - \mu_r}\Delta^2_{xy}\left({}_kV_z\right) + {}_{k-1}V_s = {}_{k+1}V_s \qquad (13)$$

### *B. Absorbing boundary conditions.*

When modelling the propagation in an unbounded medium we need to add an artificial absorbing boundary condition in the numerical limits of the simulation domain. This is achieved by introducing, in the limiting nodes, a reflection coefficient, which matches the admittances. So, different values are needed for the YZ and YX planes. The effectives relatives permittivities are:

$$\varepsilon_{xef} = \frac{\mu_r - m\varepsilon_x\beta^2}{m\varepsilon_x - \mu_r\beta^2},\ \varepsilon_{yef} = \frac{\mu_r - m\varepsilon_x\beta^2}{m\varepsilon_y(1-\beta^2)} \qquad (14)$$

The calculation of the reflection coefficients in those nodes follows the same general sketch than in the TLM modelling of anisotropic media, [2].

## III. RESULTS

We have simulated the wave propagation in uniformly moving media, with respect to the electromagnetic source, using a 2D TLM model.

### *A. Doppler effect*

The solution of every wave equation is a combination of characteristic waves with different phase velocities, corresponding to the propagation forward and backward. Then, the propagation is equivalent to the superposition of waves with speeds depending on the propagation direction, [3]–[4]. A change of the frequency occurs when a wave source is in motion respect to an observer. The frequency detected by the observer is higher/lower than that for a stationary source, when the source moves towards/away from the observer. This is the Doppler effect.

In the Fig. 1 we find the situation of the propagation of $H_z$ along the X–axis at a time enough to visualize the different wavelength dependent on the direction of propagation (forward and backward). The medium has as a diagonal permittivity tensor with $\varepsilon_x = 3.44$ and $\varepsilon_y = 1.44$, a relative permeability equal to unity and the relative velocity of the source is $\beta = 0.5$.

### *B. Wave fronts in moving isotropic media*

In an isotropic medium in rest, with respect to the source, the wave fronts are concentric circles around the point source, expanding outward. When there is relative movement, the wave fronts are no longer circles and moreover, the origin of each wave front is dragged. The fronts form now a set of ellipses with the source at one of the focus.

A time domain analysis has been performed, in order to verify the TLM results. They are shown in the Fig. 2, with $\varepsilon_r\mu_r = 1.44$ and $\beta$ values as indicated. The source is located at the cross point of the lines, and the excitation is a harmonic signal.

### *C. Wave fronts in moving anisotropic dielectric media*

In an anisotropic medium at rest, with respect to the source, the wave fronts are elliptical centered at the point source. If the electromagnetic source moves, these wave fronts are no longer elliptical, as the Fig. 3 shown for a non–magnetic medium with the diagonal permittivity tensor: $\varepsilon_x = 3.44$, $\varepsilon_y = 1.44$.

In other cases the permittivity tensor of the crystal is non–diagonal along the X– and Y– axes ($\varepsilon_{xy}$ non–zero) of the TLM network. Then, for the frame at rest, the wave fronts are turned elliptical fronts. The effect of the relative motion is a distortion due the drag of the medium. The Fig. 4 shows the TLM simulation for a permittivity tensor: $\varepsilon_x = 3.0$, $\varepsilon_y = 2.0$ and $\varepsilon_{xy} = 0.5$.

## IV. CONCLUSION

We have simulated the problem of electromagnetic wave propagation in anisotropic moving media. The simplest case of uniform movement has been considered. After the visualization of the distortion of the wave fronts in such situation and the different values of propagation speed dependent on the direction, the validity of technique is verified.

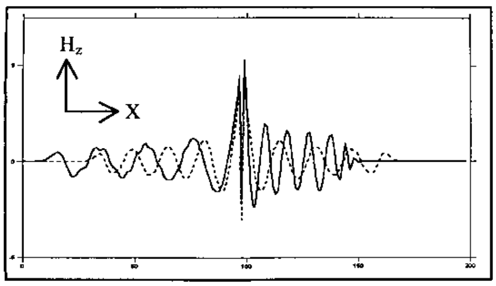


Fig. 1. The effective wavelength is greater / less than that in an stationary medium (dashed line) at right / left of the moving electromagnetic source.

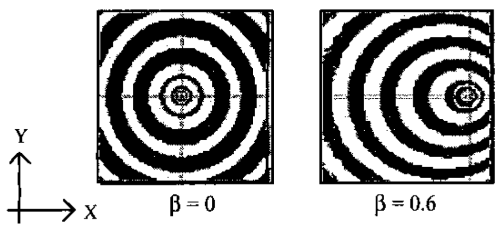


Fig. 2. Wave fronts for an isotropic medium of refraction index n = 1.2 and relative velocities $\beta$ of motion as indicated, after several iterations of the algorithm.

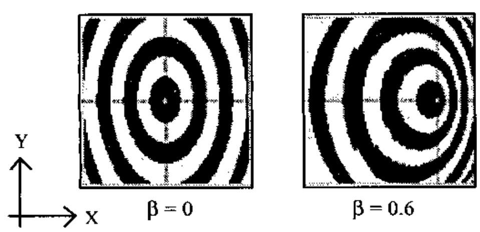


Fig. 3. Wave fronts for an anisotropic dielectric medium with permittivities $\varepsilon_x = 3.44$, $\varepsilon_y = 1.44$ and $\varepsilon_{xy} = 0.0$ and relative velocities $\beta$ as indicated, after several iterations of the algorithm.

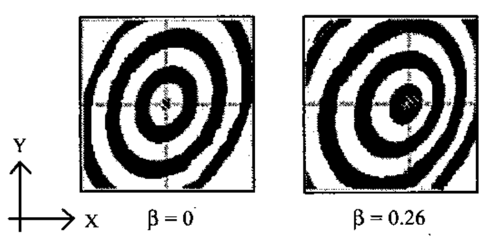


Fig. 4. Wave fronts for an anisotropic dielectric medium with permittivities $\varepsilon_x = 3.0$, $\varepsilon_y = 2.0$ and $\varepsilon_{xy} = 0.5$ and relative velocities $\beta$ as indicated, after several iterations of the algorithm.


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